



線形代数及微分I 宿題7(1)よりNo.2 解答 (9/30)

1

$$A \xrightarrow{\textcircled{3} + \textcircled{1} \times (-1)} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{pmatrix} \xrightarrow{\textcircled{3} + \textcircled{2} \times (-1)} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\textcircled{1} + \textcircled{2} \times (-1)} \begin{pmatrix} 1 & 0 & 4 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank}(A) = 2.$$

2

拡大係数行列
を簡約化する

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & 3 & 2 \\ -1 & -2 & 3 & -2 & 1 \\ 3 & 4 & 1 & 4 & 5 \end{array} \right) \xrightarrow{\textcircled{1} \times 1/2} \left(\begin{array}{cccc|c} 1 & 3/2 & -1/2 & 3/2 & 1 \\ -1 & -2 & 3 & -2 & 1 \\ 3 & 4 & 1 & 4 & 5 \end{array} \right)$$

$$\xrightarrow{\textcircled{2} + \textcircled{1}, \textcircled{3} + \textcircled{1} \times (-3)} \left(\begin{array}{cccc|c} 1 & 3/2 & -1/2 & 3/2 & 1 \\ 0 & -1/2 & 5/2 & -1/2 & 2 \\ 0 & -1/2 & 5/2 & -1/2 & 2 \end{array} \right) \xrightarrow{\textcircled{3} + \textcircled{2}, \textcircled{2} \times (-2)} \left(\begin{array}{cccc|c} 1 & 3/2 & -1/2 & 3/2 & 1 \\ 0 & 1 & -5/2 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{\textcircled{1} - \textcircled{2} \times 3/2} \left(\begin{array}{cccc|c} 1 & 0 & 7/2 & 3/2 & 7 \\ 0 & 1 & -5/2 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \text{すなわち} \quad \begin{cases} x_1 + 7x_3 = 7 \\ -x_2 - 5x_3 + x_4 = -4 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 7 - 7c_1 \\ -4 + 5c_1 - c_2 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ 0 \\ 0 \end{pmatrix} + c_1 \begin{pmatrix} -7 \\ 5 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

(c_1, c_2 : 任意定数)