



1 略。(教科書 p43 E 見よ)

2. (1) (第1行)  $3 \begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}$ , (第2行)  $-2 \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} - \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$

(2) (第2列)  $-2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix}$ , (第3列)  $-\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} + 2 \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix}$

(3) (第3行)  $-2 \begin{vmatrix} -1 & 2 & 1 \\ -1 & 2 & 2 \\ 3 & -1 & 4 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 0 & -1 & 4 \end{vmatrix} - \begin{vmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \\ 0 & 3 & 4 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 2 \\ 0 & 3 & -1 \end{vmatrix}$

(第2列)  $\begin{vmatrix} 2 & 2 & 2 \\ -2 & -1 & 2 \\ 0 & -1 & 4 \end{vmatrix} - \begin{vmatrix} 1 & 2 & 1 \\ -2 & -1 & 2 \\ 0 & -1 & 4 \end{vmatrix} - 3 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 0 & -1 & 4 \end{vmatrix} + 3 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ -2 & -1 & 2 \end{vmatrix}$

3. (1) -5 (2) -4A (3) -5

4. (1)  $\begin{vmatrix} 1 & 5 & 3 \\ 1 & 3 & 1 \\ -2 & -5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 5 & 3 \\ 0 & -2 & -2 \\ 0 & 5 & 9 \end{vmatrix} = \begin{vmatrix} -2 & -2 \\ 5 & 9 \end{vmatrix} = -8$  (2)  $\begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 0 & 0 & -1 & -2 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & -2 \end{vmatrix} = \begin{vmatrix} 0 & -1 & -2 \\ -1 & 2 & -1 \\ -1 & -1 & -2 \end{vmatrix}$   
 $= -(-1) \begin{vmatrix} -1 & -2 \\ -1 & -2 \end{vmatrix} - \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} = -(1+4) = -5$

5.  $|A_{11}| = \begin{vmatrix} -2 & 2 \\ -1 & 0 \end{vmatrix} = 2$ ,  $|A_{12}| = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2$ ,  $|A_{13}| = \begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = 1$ ,  $|A_{21}| = \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 1$   
 $|A_{22}| = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$ ,  $|A_{23}| = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = 0$ ,  $|A_{31}| = \begin{vmatrix} -1 & 1 \\ -2 & 2 \end{vmatrix} = 0$ ,  $|A_{32}| = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$ ,  $|A_{33}| = \begin{vmatrix} 1 & -1 \\ -1 & -2 \end{vmatrix} = -1$   
 余因子行列  $\tilde{A} = \begin{pmatrix} |A_{11}| & -|A_{21}| & |A_{31}| \\ -|A_{12}| & |A_{22}| & -|A_{32}| \\ |A_{13}| & -|A_{23}| & |A_{33}| \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 \\ -2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}$ ,  $|A| = |A_{11}| + |A_{12}| + |A_{13}| = 1$

ゆえ  $A^{-1} = \frac{1}{|A|} \tilde{A} = \begin{pmatrix} 2 & -1 & 0 \\ -2 & -1 & -1 \\ 1 & 0 & -1 \end{pmatrix}$

6. (1)  $abcd + 2b + 1$  (2)  $a^4 - 3a^2 + 1$  (3)  $(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)$

7.  $\begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_2 & x_3 & x_4 & x_1 \\ x_3 + x_4 & x_4 + x_1 & x_1 + x_2 & x_2 + x_3 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_2 & x_3 & x_4 & x_1 \\ x_1 + x_2 + x_3 + x_4 & x_2 + x_3 + x_4 + x_1 & x_3 + x_4 + x_1 + x_2 & x_4 + x_1 + x_2 + x_3 \end{vmatrix}$   
 (第2行と第3行を第4行にたす)

$= (x_1 + x_2 + x_3 + x_4) \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ x_2 & x_3 & x_4 & x_1 \\ 1 & 1 & 1 & 1 \end{vmatrix} = 0$  (⊖) 同1行がある.)

9. 略。

8. (1)  $X$  が正則より  $X^{-1}$  が存在  $\det I = \det(X X^{-1}) = \det(X) \det(X^{-1})$  より  $\det(X^{-1}) = \frac{1}{\det(X)}$  (2)  $X^k = I$  より  $\det(X^k) = (\det(X))^k = (\det(I))^k = 1$ .

$X$  が実の行列より  $k$  が奇数のとき  $\det(X) = 1$ , 偶数のとき  $\det(X) = 1$ ,  $k \leq 0$   $\det(X) = -1$