



線形代数及V-演習II 宿題701121-10.5 解答 (10.28)

1.

$$A = (a_1, a_2, a_3, a_4) = \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & -4 \\ 3 & 0 & 3 & 7 \\ 0 & -1 & -2 & 0 \end{pmatrix} \text{ とおいて簡約化する}$$

$$A \rightarrow \begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & -3 \\ 0 & -3 & -6 & 10 \\ 0 & -1 & -2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{rank}(A) = 3$ より 最大個数は3.

$$\begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \text{ より}$$

$$a_3 = -a_1 + 2a_2$$

2. $f_1 = (1 \ x \ x^2 \ x^3) \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, f_2 = (1 \ x \ x^2 \ x^3) \begin{pmatrix} 2 \\ 3 \\ 0 \\ -1 \end{pmatrix}, f_3 = (1 \ x \ x^2 \ x^3) \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}$

$$f_4 = (1 \ x \ x^2 \ x^3) \begin{pmatrix} 2 \\ 6 \\ -2 \\ -1 \end{pmatrix} \text{ とおけるので}$$

$$\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ -2 \\ -1 \end{pmatrix} \text{ を基底とする}$$

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 3 & -1 & 6 \\ 0 & 0 & 2 & -2 \\ 0 & -1 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & -1 & -3 & 2 \\ 0 & 0 & 2 & -2 \\ 0 & -1 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{rank}(A) = 3 \quad \text{最大個数は3}$$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \text{ より } f_4 = f_1 + f_2 - f_3$$