Workshop on
Nonlinear Partial Differential Equations
― Japan-China Joint Project for Young Mathematicians 2018 ―

Program & Abstract

October 26-28, 2018

at Ryukoku University

Presentation Room, Building 6, Seta Campus

October 26 (Friday)

9:55 – 10:00   **Opening**

10:00 – 10:45   **Zhitao Zhang** (The Chinese Academy of Sciences)
Some new results on Henon-Lane-Emden conjecture and
Schrödinger systems

11:00 – 11:45   **Kanako Suzuki** (Ibaraki University)
Stability and blowup of solutions to some reaction-diffusion-ODE
systems

12:00 – 13:30   **Lunch**

13:30 – 14:00   **Lorenzo Cavallina** (Tohoku University)
Local analysis of the torsional rigidity of a two-phase body
under general perturbations

14:05 – 14:35   **Zengyun Qin** (East Chian Normal University)
Asymptotic behavior of vector fields in cylinders and perforated
domains

14:45 – 15:30   **Tatsuki Mori** (Osaka University)
Numerical analysis of multiplicity and stability of solutions
for the SKT cross-diffusion stationary limiting equation

15:30 – 15:50   **Break**

15:50 – 16:35   **Xia Huang** (East China Normal University)
On the Conformal Metrics with Prescribed Q-curvature
in Higher Dimensions

16:45 – 17:30   **Xingfei Xiang** (Tongji University)
On an $H^r(\mathrm{curl}, \Omega)$ estimate for a Maxwell type system
in convex domains

19:00 –   **Party at "Rakuza"**

October 27 (Saturday)

10:00 – 10:45   **Takiko Sasaki** (Meiji University)
The blow-up curve of solutions for semilinear wave equations
with Dirichlet boundary conditions in one space dimension

11:00 – 11:45   **Huirong Pi** (Guangxi University)
Existence results for a Kirchhoff equation with a general
nonlinear term

12:00 – 13:30   **Lunch**

13:30 – 14:00   **Asato Mukai** (University of Tokyo)
Large time behavior of solutions of the heat equation
with inverse square potential

14:15 – 15:00   **Huyuan Chen** (Jiangxi Normal University)
On isolated singularities for elliptic equation with Hardy operator

15:15 – 16:00   **Takashi Kagaya** (IMI, Kyushu University)
On traveling waves for one-dimensional surface diffusion
with contact angle condition

16:00 – 16:05   **Closing**

16:10 –   **Free discussions**

October 28 (Sunday)

——— **Free discussions** ———

Organizers:
Tatsuki Kawakami (Ryukoku University)
Yoshihisa Morita (Ryukoku University)
Xingbin Pan (East China Normal University)
Feng Zhou (East China Normal University)

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Some new results on Henon-Lane-Emden conjecture and Schrödinger systems

Zhitao Zhang (The Chinese Academy of Sciences)

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We prove Henon-Lane-Emden conjecture is true for space dimension $N = 3$. We prove some new results on some Schrödinger systems: symmetry and asymptotic behavior of ground state solutions; bifurcation and multiple existence of positive solutions; uniqueness and structure of positive solutions; symmetry breaking via Morse index; limit configurations of Schrödinger systems versus optimal partition for the principal eigenvalue of elliptic systems. We also give Dancer-Fucik spectrum for fractional Schrödinger operators with a steep potential well on $\mathbb{R}^3$.

Stability and blowup of solutions to some reaction-diffusion-ODE systems

Kanako Suzuki (Ibaraki University)

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We discuss the following system for unknown functions $u = u(x, t)$ and $v = v(x, t)$:

$$u_t = f(u, v), \qquad \text{for} \quad x \in \overline{\Omega}, \ t > 0,$$
$$v_t = D\Delta v + g(u, v), \quad \text{for} \quad x \in \Omega, \ t > 0, \frac{\partial v}{\partial \nu} = 0 \quad \text{for} \quad x \in \partial\Omega, \ t > 0,$$

where, $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, ν is the outer unit normal to $\partial\Omega$ and $D > 0$ is a constant which denotes a diffusion coefficient for v.

The systems of a single reaction-diffusion equation coupled with an ordinary differential equation are often used as models of pattern formation phenomena. Such system arise, for example, from modeling of interactions between cellular processes such as cell growth, differentiation or transformation and diffusion signaling factors.

In this talk, we would like to understand the stability of stationary solutions and the dynamics of reaction-diffusion-ODE systems. We can show that all spatially heterogeneous regular stationary solutions are unstable. This implies that there is no stable spatial pattern, which is very different from classical reaction-diffusion equations modeling pattern formation. Concerning the dynamics, we see that space inhomogeneous solutions become unbounded in either finite or infinite time, even if space homogeneous solutions are bounded uniformly in time.

These are joint works with A. Marciniak-Czochra (University of Heidelberg) and G. Karch (University of Wroclaw).

Local analysis of the torsional rigidity of a two-phase body under general perturbations

Lorenzo Cavallina (Tohoku University GSIS)

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We consider the abstract mathematical model of a (two-phase) composite medium containing a core made of a dierent material. We study the so-called torsional rigidity of this object. Our aim is to investigate how rotational symmetry is related to optimality (in the precise sense of "maximizing the torsional rigidity functional under volume constraints"). We will perform shape derivatives up to the second order, with respect to general perturbations of our structure, to discover a symmetry breaking phenomenon.

Asymptotic behavior of vector fields in cylinders and perforated domains

Zengyun Qin (East China Normal University)

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The first part of this talk concerns the boundary value problem of the time-independent Maxwell equations in a cylinder with variable height. Assuming the boundary datum is independent of the axial coordinate, we show that, as

the height of the cylinder increases to infinity, the solution locally converges exponentially fast to a solution of the Maxwell equations in the cylinder with infinite height. In the second part of this talk, we discuss solutions of the div-curl system in a perforated domain, with many small balls deleted. Assuming the capacity of the deleted balls tends to zero, we describe the asymptotic behavior of the solutions. The Dirichlet fields and Neumann fields on the perforated domain are also examined. The second part is a joint work with Prof. Xingbin Pan.

Numerical analysis of multiplicity and stability of solutions for the SKT cross-diffusion stationary limiting equation

Tatsuki Mori (Osaka University)

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We are interested in SKT cross-diffusion model proposed by N. Shigesada, K. Kawasaki and E. Teramoto in 1979. This model describes segregation phenomena of two competing species in the same habitat area. The effect of cross-diffusion affects the population pressure between two different kinds. It is an interesting problem to see whether this effect may give rise to a spatial segregation or not. Y. Lou and W.-M. Ni derived limiting equations of this model to clarify this problem. It has been thought that the number of solutions of a stationary limiting equation seems to be at most two. However, we have found several parameter values numerically for which there exist three solutions. In this talk, we show numerical overviews of existence, non-existence, multiplicity and stability of solutions to the stationary limiting equation.

On the Conformal Metrics with Prescribed Q-curvature in Higher Dimensions

Xia Huang (East China Normal University)

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We will talk about the Q -curvature equation $(-\Delta)^n u = K(x)e^{2nu}$ in $\mathbb{R}^{2n}$. When $n = 1$, it is the prescribed Gauss curvature equation $-\Delta u = K(x)e^{2u}$ in $\mathbb{R}^2$, called also the Liouville equation, for which many classical results exist. For $n \geq 2$, the case $K(x) \equiv$ constant was studied a lot. But there are few works for non constant curvature case. Here we will try to present some recent results, for both the constant and non constant curvature cases for higher dimensions. This is a joint work with Professors D. Ye and F. Zhou.

On an $H^r(\text{curl}, \Omega)$ estimate for a Maxwell type system

Xingfei Xiang (Tongji University)

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Abstract: In this talk we first review the estimates for vector fields involving the operators div and curl in smooth domains and in Lipschitz domains. Then we show the estimates in bounded convex domains with the boundary data belonging to L^r space. At last, as an application of the estimate, we consider an $H^r(\text{curl}, \Omega)$ estimate for solutions to a Maxwell type system with an inhomogeneous boundary condition in convex domains.

The blow-up curve of solutions for semilinear wave equations with Dirichlet boundary conditions in one space dimension

Takiko Sasaki (Meiji University)

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We study a blow-up curve for the one dimensional wave equation $\partial_t^2 u - \partial_x^2 u = 2^p|\partial_t u|^p$. In what follows, we call this equation (1). Blow-up curves have been studied from the viewpoints of its differentiability and singularity. Merle and Zaag (2012) considered $\partial_t^2 u - \partial_x^2 u = |u|^p$. They showed that the blow-up curve has singular points if initial conditions have sign changes. They used the variational structure of the wave equations to prove the singularity of the blow-up curve. In the case equation (1), it is not clear for the author when

the blow-up curve has the singular points because the equation (1) does not have any variational structure. In this talk, we consider the blow-up curve for the equation (1) and show the blow-up curve has sign changes under suitable initial conditions.

Existence results for a Kirchhoff equation with a general nonlinear term

Huirong Pi (Guangxi University)

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In this talk, we will first review the known results of Kirchhoff equation with superlinear nonlinearity and then consider a Kirchhoff equation with the non-linear term asymptotically linear at infinity. This problem involves a nonlocal term which arises in several physical and biological systems and causes some mathematical difficulties. Using a variational approach, we prove the existence of a positive solution by mininization on a new manifold, and finally give the asymptotic behavior of solutions when the parameter of nonlocal term goes to zero.

Large time behavior of solutions of the heat equation with inverse square potential

Asato Mukai (University of Tokyo)

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This talk is based on a joint work with Professor Ishige. Consider the following problem

(P) $\qquad \partial_t u - \Delta u + V(|x|)u = 0$ in $\mathbb{R}^N \times (0,\infty), \quad u(x,0) = \varphi(x)$ in $\mathbb{R}^N$,

where $N \geq 2$, $\varphi \in L^2(\mathbb{R}^N, e^{-|x|^2/4}\,dx)$ and $V = V(r)$ is a inverse square potential, that is,

(V) $\begin{cases} \text{(i)} \quad V \in C^1((0,\infty)); \qquad \text{(ii)} \quad \sup_{r \geq 1}|r^3 V'(r)| < \infty; \\ \text{(iii)} \quad \lim_{r \to 0} r^{-\theta}\,|r^2 V(r) - \lambda_1| = 0, \quad \lim_{r \to \infty} r^\theta\,|r^2 V(r) - \lambda_2| = 0, \\ \qquad \text{for some } \lambda_1, \lambda_2 \in [\lambda_*, \infty) \text{ with } \lambda_* := -(N-2)^2/4 \text{ and } \theta > 0. \end{cases}$

The purpose of this study is to establish a method for obtaining the large time behavior of $e^{-tL}\varphi$ with $L := -\Delta + V$. We define the **non-negativity** and **criticality** of an operator.

Definition (non-negativity). We say $L := -\Delta + V$ is nonnegative on $L^2(\mathbb{R}^N)$ if

$$(L\phi, \phi)_{L^2(\mathbb{R}^N)} = \int_{\mathbb{R}^N} [|\nabla \phi|^2 + V(|x|)\phi^2]\, dx \geq 0, \quad \phi \in C_0^\infty(\mathbb{R}^N \setminus \{0\}).$$

Definition (criticality). When L is nonnegative, we say that

- L is **subcrutical** if $Lu = 0$ in $\mathbb{R}^N$ has a positive Green function;

- L is **critical** if L is not subcritical.

On the other hand, L is supercritical if L is not nonnegative.

Example. $-\Delta$ is subcritical when $N \geq 3$ and critical when $N = 2$.

We consider the following ordinary differential equation

$$\text{(O)} \qquad U'' + \frac{N-1}{r} U' - V(r)U = 0 \text{ in } (0, \infty), \quad U(r) \sim r^{A^+(\lambda_1)} \text{ as } r \to 0,$$

under the condition (V). When L is nonnegative, equation (O) has a positive solution U which satisfies

$$U(r) \sim \begin{cases} c_* r^{A^+(\lambda_2)} & (L \text{ is subcritical and } \lambda_2 > \lambda_*), \\ c_* r^{-\frac{N-2}{2}} \log r & (L \text{ is subcritical and } \lambda_2 = \lambda_*), \\ c_* r^{A^-(\lambda_2)} & (L \text{ is critical}), \end{cases}$$

as $r \to \infty$, where $c_* > 0$ and $A^\pm(\lambda) := \dfrac{-(N-2) \pm \sqrt{(N-2)^2 + 4\lambda}}{2}$. (See [3], [5].)

Definition. When L is critical, we say that

- L is **null-critical** if $U \notin L^2(\mathbb{R}^N)$ (i.e. $A^-(\lambda_2) \geq -N/2$);

- L is **positive-critical** if $U \in L^2(\mathbb{R}^N)$ (i.e. $A^-(\lambda_2) < -N/2$).

In subcritical case, Ishige and Kabeya studied with some additional restrictions such as $V \in C([0, \infty))$, $\lambda_2 > \lambda_*$ and the sign of the potential (see [1]). On the other hand, in null-critical case, we only know time decay of the fundamental solution. The main aim of this talk is to establish a method for obtaining the precise description of the large time behavior of $e^{-tL}\varphi$ in the subcritical case and in the null-critical case with $A^-(\lambda_2) > -N/2$.

Theorem. Let $N \geq 2$ and $\varphi \in L^2(\mathbb{R}^N, e^{|x|^2/4} \, dx)$. Let u be a solution of (P) under the condition (V). When L is subcritical or null-critical with $A^-(\lambda_2) > -N/2$,

$$\lim_{t \to \infty} t^{\frac{N+A}{2}} u(t^{\frac{1}{2}} y) = M(\varphi)|y|^A e^{-|y|^2/4}$$

uniformly on $\{x \in \mathbb{R}^N \mid K^{-1} t^{1/2} \leq |x| \leq K t^{1/2}\}$ for any $K \geq 1$ and

$$t^{\frac{N+2A}{2}} \frac{u(x)}{U(|x|)} = c_*^{-1} M(\varphi) + t^{-1} O(|x|^2) + o(1) \text{ as } t \to \infty$$

uniformly on $B(0, \epsilon(1+t)^{1/2})$ for any sufficiently small $\epsilon > 0$, where

$$A := \begin{cases} A^+(\lambda_2) & (L \text{ is subcritical}), \\ A^-(\lambda_2) & (L \text{ is critical}), \end{cases} \quad M(\varphi) := \frac{1}{c_* \kappa} \int_{\mathbb{R}^N} \varphi(x) U(|x|) \, dx \text{ and } \kappa > 0.$$

Outline of Proof. Let u be a radially symmetric with respect to x. In the paper [1], we apply the forward similar transformation, that is,

$$v(\xi, s) := (1+t)^{\frac{N}{2}} u(|x|, t) \text{ with } \xi := (1+t)^{-\frac{1}{2}} |x| \text{ and } s := \log(1+t).$$

This v satisfies the N-dimensional parabolic equation. Next, we consider the eigenvalue problem. However, the first eigenfunction does not belong to $H^1(\mathbb{R}^N)$ in null-critical case.

To overcome this problem, we apply the **weighted** forward similar transformation, that is,

$$w(\xi, s) := (1+t)^{\frac{d}{2}} \underline{|x|^{-A}} u(|x|, t) \text{ with } \xi := (1+t)^{-\frac{1}{2}} |x| \text{ and } s := \log(1+t)$$

where $d := N + 2A > 0$. This w satisfies the following "d-dimentional" parabolic equation

$$\partial_t w = \frac{1}{r^{d-1}} \partial_r (r^{d-1} \partial_r w) - V_{\lambda_2}(r) w \text{ in } (0, \infty) \times (0, \infty)$$

where $V_{\lambda_2}(r) := V(r) - \lambda_2 r^{-2} = o(r^{-2})$ as $r \to \infty$. We remark that d is not necessarily natural number.

References

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On isolated singularities for elliptic equation with Hardy operator

Huyuan Chen (Jiangxi Normal University)

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In this talk, we would like to talk about the isolated singularities for Hardy operators: 1. the fundamental solutions of Hardy operator

$$\mathcal{L}_\mu := -\Delta - \frac{\mu}{|x|^2};$$

2. qualitative properties of the solution of nonhomogeneous of

$$\mathcal{L}_\mu u = f \quad \text{in} \quad \Omega \setminus \{0\}, \qquad u = 0 \ \text{ on } \ \partial\Omega;$$

3. the classification, existence and nonexistence of the isolated singularities of semilinear Hardy equation

$$\mathcal{L}_\mu u = u^p \quad \text{in} \quad \Omega \setminus \{0\}, \qquad u = 0 \ \text{ on } \ \partial\Omega.$$

4. Application to semilinear linear elliptic equation $-\Delta u = V f(u)$ in exterior domain.

On traveling waves for one-dimensional surface diffusion with contact angle condition

Takashi Kagaya (IMI, Kyushu University)

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In this talk, we deal with an evolving plane curve with two endpoints, which can move freely on the x-axis with generating constant contact angles. For the evolution of this plane curve governed by surface diffusion, we discuss the existence, the uniqueness and the convexity of traveling waves. The main results is that the uniqueness and the convexity can be lost in depending on the conditions of the contact angles, although the existence holds for any contact angles in the interval $(0, \pi/2)$. Our strategy consists of the shooting method and the analysis of "oscillation" properties for the solution to a 3rd order ordinary differential equation. As a result, we can construct traveling waves of which profile curves are represented by "oscillating" graphs if one of the contact angles is sufficiently small. This talk is based on a joint work with professor Yoshihito Kohsaka (Kobe university).