Abstracts

Long-time dynamics for some energy critical heat flows

Yifu Zhou
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In this talk, we shall briefly introduce some recent gluing constructions of long-time trichotomy dynamics for the harmonic map heat flow with 1-equivariant symmetry and the energy critical Fujita equations. Depending on the initial data in a precise manner, these global solutions can exhibit blow-up, blow-down or remain bounded at time infinity, motivated mainly by a program proposed by M. Fila and J.R. King. This talk is based on joint works with Z. Li, J. Wei, and Q. Zhang.

Remarks on the asymptotic behavior of solutions to the six dimensional Fujita equation

Junichi Harada
Akita University

We discuss the asymptotic behavior of solutions to the 6D Fujita equation: $u_t = \Delta u + |u|u$. It is known that this equation has a type II blowup solution whose energy is infinite. We here give some remarks on the possibilities of a type II blowup with finite energy.

A relation between a solution to Hénon type equation and initial data

Yusuke Oka
The University of Tokyo

We consider the Cauchy problem for an Hénon type equation

$$\begin{cases} \partial_t u = \Delta u + |x|^\beta |u|^{\gamma-1} u, & x \in \mathbb{R}^N,\ t \in \mathbb{R}_{>0}, \\ u(x,0) = \mu(x) \in \mathcal{S}'(\mathbb{R}^N), & x \in \mathbb{R}^N, \end{cases} \tag{P}$$

where $\beta \geq 0$, $\gamma > 1$ and $N \in \mathbb{Z}_{\geqslant 1}$. In this talk, we concentrate on weighted Bochner spaces $L^q(\mathbb{R}_{>0}; L^p_\alpha(\mathbb{R}^N))$ for classes of solutions to the problem (P), where L^p_α denotes weighted Lebesgue spaces and the exponents satisfy the scale invariant condition $\frac{\beta+2}{\gamma-1} = \frac{N}{p} + \frac{2}{q} + \alpha$. I explain that (homogeneous) weighted Besov spaces $\dot{B}^{-2/q;\alpha}_{p,q}(\mathbb{R}^N)$ arise reasonably as classes of initial data $\mu(x)$. This work is inspired by Kozono–Okada–Shimizu(2020) and Chikami–Ikeda–Taniguchi(2022).

Classification of bifurcation diagrams for semilinear elliptic equations

Kenta Kumagai
Tokyo Institute of Technology

In this talk, we consider the global bifurcation diagram of radial solutions for the semiliear elliptic problem

$$-\Delta u = \lambda a(|x|)e^u \ \text{ in } B_1, \quad u > 0 \ \text{ in } B_1, \quad u = 0 \ \text{ on } \partial B_1, \tag{G}$$

where $\lambda > 0$ is a parameter and $B_1 \subset \mathbb{R}^N (N \geq 3)$ is the unit ball. We assume that the weight $a : [0,1] \to \mathbb{R}$ satisfies $a(|x|) \in C^2(\overline{B_1})$, $a(r) > 0$, $a(0) = 1$.

When $a = 1$, the set of radial solutions of (G) is an unbounded curve described as $(\lambda(\alpha), u(r,\alpha))$, where $\alpha := \|u\|_{L^\infty(B_1)}$ and $r := |x|$. Moreover, it is well-known that the bifurcation structure changes depending on the dimension N. In fact, Joseph and Lundgren [2] proved the following: when $N \leq 9$, the bifurcation curve turns infinitely many times around some λ_* (Type I). On the other hand, when $N \geq 10$, the bifurcation curve is parametrized by λ. Moreover, $\alpha(\lambda)$ is increasing and it blows up at some λ_* (Type II). Initiated by this result, many attempts have been done in order to determine the bifurcation structure for various nonlinearities f. We point out that Miyamoto [1] provided an example of f for which the bifurcation curve turns at least one but finitely many times (Type III) for a sufficiently large N.

Our goal is to give an answer to the following fundamental question: does the perturbation of the weight a affect the bifurcation structure in the critical dimension $N = 10$? In this talk, we give a positive answer to the question. Moreover, we give an optimal classification of the bifurcation structure.

Theorem 1. *The bifurcation diagram of* (G) *is of*

(i) Type I if $3 \leq N \leq 9$,

(ii) Type II if $N = 10$ *and* $(a/a_H)' \leq 0$ *in* $(0,1]$,

(iii) Type III if $N = 10$ *and* $(a/a_H)' > 0$ *in* $(0,1]$,

where H *is the first eigenvalue of* $-\Delta_D$ *in the* 2*-dimensional unit ball and*

$$a_H(r) := \left(1 + \frac{H}{2(N-2)}r^2\right) e^{\frac{H}{2N}r^2}.$$

Our strategy is to find the specific radial singular solutions U_h for the specific weight a_h with $h \geq 0$ and study the stability of U_h. We remark that the improved Hardy inequality plays an important role in the stability of U_h.

[1] Y. Miyamoto, *Structure of the positive solutions for supercritical elliptic equations in a ball*, J. Math. Pures Appl. (9) **102** (2014), no. 4, 672–701.

[2] D. D. Joseph and T. S. Lundgren, *Quasilinear Dirichlet problems driven by positive sources*, Arch. Rational Mech. Anal. **49** (1972/73), 241–269.

Properties of a solution to a free boundary problem with flux boundary conditions and its application to two-scale models

Kota Kumazaki
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In this talk, we consider a free boundary problem with a flux boundary condition. This problem is proposed as a mathematical model describing water filling phenomenon in one microscopic pore and consists of a diffusion equation for water content and an ordinary differential equation describing the speed of the front of the water region. In addition, flux boundary conditions based on the mass conservation law are imposed on fixed boundaries and moving boundaries. In this talk, we discuss the solvability of the problem, continuous dependence of the solution, and large-time behavior. Also, we introduce an application of these results to a mathematical model based on the concept of two-scale.

Critical norm blow-up for a supercritical semilinear heat equation

Jin Takahashi
Tokyo Institute of Technology

We consider the scaling critical Lebesgue norm of blow-up solutions to a supercritical semilinear heat equation (the Fujita equation). We show that the critical norm also blows up at the blow-up time. This talk is based on joint works with Hideyuki Miura (Tokyo Institute of Technology).

Existence of solutions for fractional semilinear parabolic equations in Besov-Morrey spaces

Erbol Zhanpeisov
OIST

In this talk, we consider the existence of solutions of semilinear parabolic equations with fractional diffusion. In the initial value problem for fractional diffusion equations with power-type nonlinear terms, the relation between the existence of time-local solutions and the optimal singularity allowed for the initial values is known in the framework of Radon measures. The purpose of this talk is to show the existence of solutions for a wide range of initial values, including derivatives of the delta function, by introducing local Besov-Morrey spaces. Furthermore, using the Sobolev type embedding theorem in Besov-Morrey spaces, we confirm the existence of solutions for initial values with the optimal singularity, and discuss the relation between the order of derivatives of the Radon measure allowed for the initial values and the power of fractional diffusion. The results are also applicable to higher-order equations and viscous Hamilton-Jacobi equations. In the first half of the talk, I will discuss known results and properties of local Besov-Morrey spaces, and in the second half, I will present the decay estimate for the heat kernel of the fractional Laplacian and the proof of the main theorem using the contraction mapping theorem.

Li–Yau type inequality for the p-bending energy

Kensuke Yoshizawa
Kyushu University

Li–Yau obtained the sharp inequality that the Willmore energy of a closed surface is bounded below by 4π times multiplicity. Its one-dimensional counterpart involving the so-called bending energy for planar curves has been also studied. In this talk, we generalize the bending energy to the p-bending energy for $p > 1$ to discuss analogous results, and then reveal some new phenomena on its optimality arising from the generality of $p > 1$. This talk is based on a joint work with Prof. Tatsuya Miura (Tokyo Institute of Technology).

Asymptotically self-similar global solutions for Hardy-Hénon parabolic equations

Noboru Chikami
Nagoya Institute of Technology

We construct global asymptotically self-similar solutions to the Hardy-H'enon parabolic equation for a large class of initial data belonging to weighted Lorentz spaces. The solution may be asymptotic to a self-similar solution of the linear heat equation or to a self-similar solution to the Hardy-H'enon equation depending on the decay of the initial data at infinity. The asymptotic results are new for the Hardy case.

Singular solutions for semilinear elliptic equations with exponential type nonlinearities in 2-d

Norisuke Ioku
Tohoku University

We consider radial singular solutions of the semilinear elliptic equation $\Delta u + f(u) = 0$ in $B_R \setminus \{0\}$, where B_R is a ball in $\mathbb{R}^2$. In this talk we focus on exponential nonlinearities as a critical growth in two dimensions and verify an exact asymptotic expansion of the singular solution. This talk is based on a joint work with Yohei Fujishima (Shizuoka University), Elide Terraneo (Milano University), and Bernhard Ruf (Milano University).